Review of Automatic Control

State Feedback

Per Mattsson

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Introduction

- The goal of control is to choose the input $u(t)$ so that the plant behaves in a desirable way.

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Introduction

$$\xrightarrow{u(t)} \boxed{G(s)} \xrightarrow{y(t)}$$

- The goal of control is to choose the input $u(t)$ so that the plant behaves in a desirable way.
- Often we want the output to follow some reference $r(t)$.

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State Feedback

Consider a state space model:

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t).$$

- The state $x(t)$ contains all information needed to predict future outputs.

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$$u(t) = -Lx(t) + L_r r(t).$$

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- Linear state feedback:

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- The designer has to determine suitable L and L_r.

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Inserting $u(t)$ into the state equation gives

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$$\dot{x}(t) = Ax(t) + B(-Lx(t) + L_r r(t)) = (A - BL)x(t) + BL_r r(t).$$

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- **Poles:** Given by the eigenvalues of $A - BL$.

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- **Closed-loop system:** $G_c(s) = C(sI - A + BL)^{-1}BL_r$.
- **Poles:** Given by the eigenvalues of $A - BL$.
- **Static gain:** $G_c(0) = C(-A + BL)^{-1}BL_r$.

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Example: State feedback

$$\dot{x}(t) = \begin{bmatrix} -2 & -1 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} x(t).$$

Let $L = \begin{bmatrix} \ell_1 & \ell_2 \end{bmatrix}$.

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The eigenvalues of $A - BL$ are given by the zeros to the characteristic polynomial

$$\det(sI - A + BL)$$

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$$\det(sI - A + BL) = \begin{vmatrix} s + 2 + \ell_1 & 1 + \ell_2 \\ -1 & s \end{vmatrix}$$

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By choosing ℓ_1 and ℓ_2 we can get any desired characteristic equation, and hence any desired poles.

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Hence, to get poles -2 and -3, and static gain 1, choose

$$u(t) = -\begin{bmatrix}3 & 5\end{bmatrix} x(t) + 6r(t).$$

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Controllability

- Remember that a state space model is controllable if and only if the controllability matrix

$$\mathcal{S} = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$$

has full rank.

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- Remember that a state space model is controllable if and only if the controllability matrix

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  has full rank.
- We can place the eigenvalues of $A - BL$ anywhere we want by choosing L *if and only if* the state space model is controllable.

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Observers

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$$y(t) = Cx(t).$$

- One problem with state feedback is that the full state $x(t)$ is typically not measured. We only measure the output $y(t)$.

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$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K(y(t) - C\hat{x}(t)).$$

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- The designer has to choose K so that the estimation error $\tilde{x}(t) = x(t) - \hat{x}(t)$ is well-behaved.

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and the observer

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The state estimation error $\tilde{x}(t) = x(t) - \hat{x}(t)$ then satisfy

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Hence, if the initial estimation error is $\tilde{x}(0)$, then

$$\tilde{x}(t) = e^{(A-KC)t}\tilde{x}(0).$$

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Observer

- We have seen that the estimation error for the observer satisfy

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- If all eigenvalues of $A - KC$ lies strictly in the left half-plane, then the error $\tilde{x}(t)$ will go to zero as $t \to \infty$.

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- The eigenvalues of $A - KC$ are called the **observer poles**.

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- The eigenvalues of $A - KC$ are called the **observer poles**.
- K can be chosen to get any desired observer poles if and only if the system is observable.

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Feedback from reconstructed state

- If an observer is used to find an estimate of $x(t)$, then we can use feedback from the reconstructed states. That is:

$$\begin{aligned} u(t) &= -L\hat{x}(t) + L_r r(t) \\ \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + K(y(t) - C\hat{x}(t)), \end{aligned}$$

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$$\xrightarrow{\quad u(t) \quad} \boxed{\text{System}} \xrightarrow{y(t)}$$

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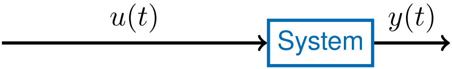
where K, L and L_r has to be designed.



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Feedback from reconstructed state

- If an observer is used to find an estimate of $x(t)$, then we can use feedback from the reconstructed states. That is:

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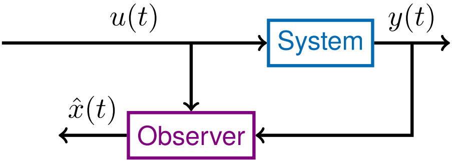

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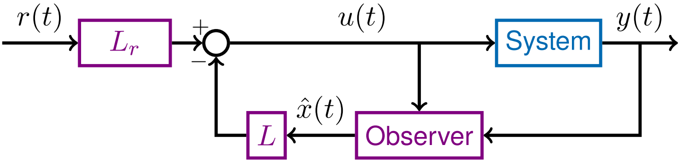

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