HÖGSKOLAN
I GÄVLE

Review of Automatic Control

Feedback

Per Mattsson

per.mattsson@hig.se

Introduction

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per.mattsson@hig.se

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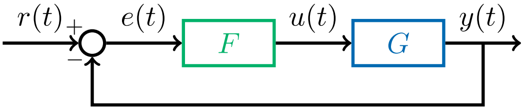
- The goal of control is to choose the input $u(t)$ so that the plant behaves in a desirable way.
- Often we want the output to follow some reference $r(t)$.
- In feedback control, the controller uses measured values of the output $y(t)$ in order to determine a suitable input signal $u(t)$.

per.mattsson@hig.se

Feedback

A common control structure is the following simple negative feedback:



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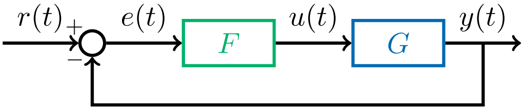

- Here the controller uses the control error $e(t) = r(t) - y(t)$ in order to determine a suitable input $u(t)$ to the plant.

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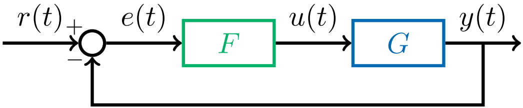

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$$G_c(s) = \frac{Y(s)}{R(s)} = \frac{F(s)G(s)}{1 + F(s)G(s)}.$$

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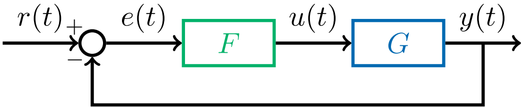

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- The behaviour of the closed-loop system depends on the choice of controller $F(s)$.

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Example: P-control

In proportional control (P-control), we let the input signal be

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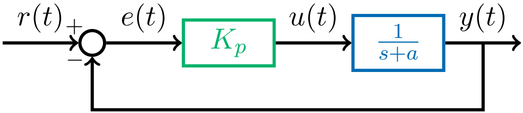

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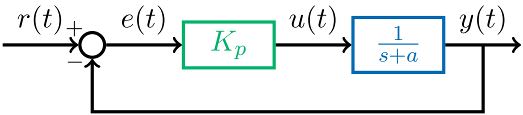

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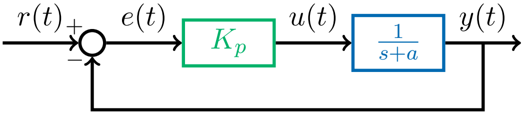

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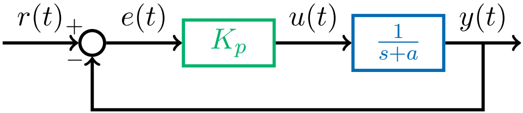

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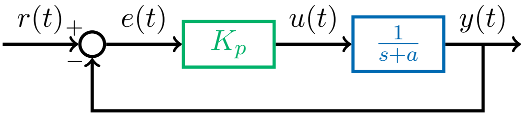

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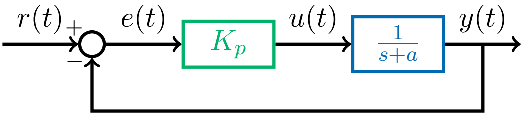

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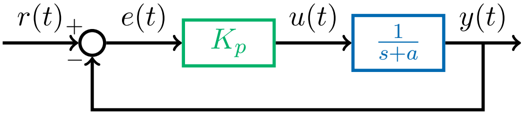

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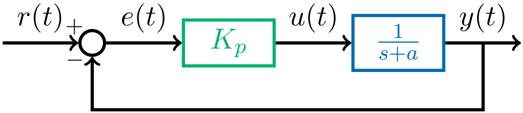

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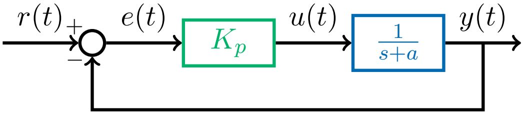

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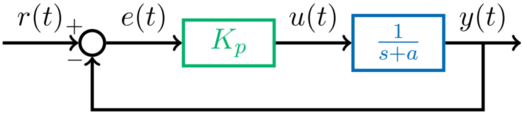

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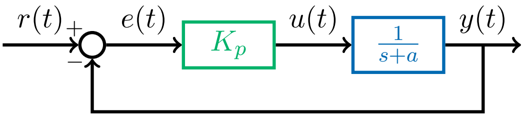

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Example: PI-control

In order to remove the static error, we can add an integrating part:

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Choose: $K_p = -p_1 - p_2 - a$, $K_i = p_1 p_2$.

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PID-controller

A very popular control structure is the PID-controllers:

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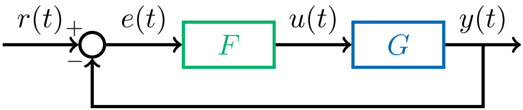

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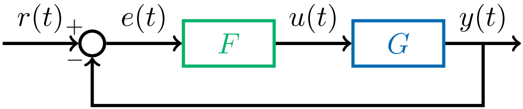

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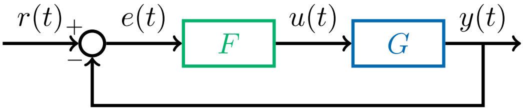

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Possible ways to choose K_p, K_i, K_d: Trail and error, rules of thumb, pole placement, studying Bode plots.

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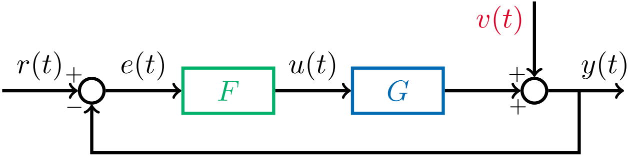
Disturbances in feedback systems



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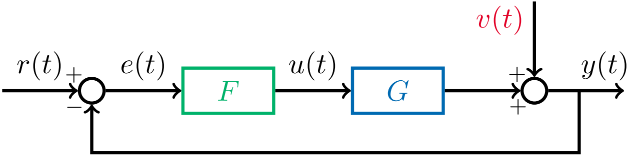

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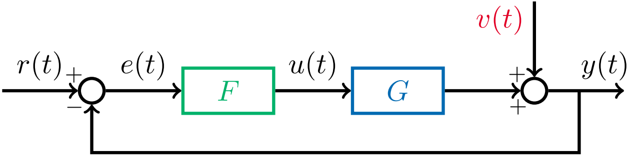

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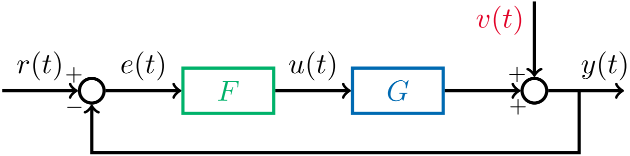

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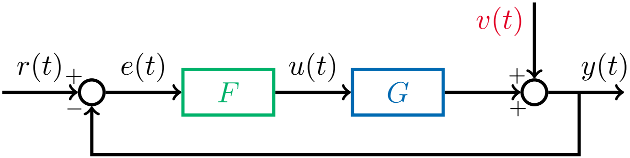
Solving for $Y(s)$ results in

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)} R(s) + \frac{1}{1 + G(s)F(s)} V(s).$$

per.mattsson@hig.se

Disturbances in feedback systems



It is also important to study how disturbances affect the system. For example, with the disturbance $v(t)$ as in the block diagram, we get

$$Y(s) = G(s)U(s) + V(s) = G(s)F(s)(R(s) - Y(s)) + V(s)$$

Solving for $Y(s)$ results in

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)F(s)}R(s) + \frac{1}{1 + G(s)F(s)}V(s).$$

The transfer function $G_c(s)$ from $R(s)$ to $Y(s)$ is the same as before.

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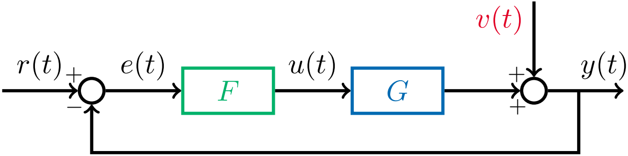

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The transfer function from $V(s)$ to $Y(s)$ is $S(s) = \dfrac{1}{1 + G(s)F(s)}$.

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