Review of Automatic Control

Step response

Per Mattsson

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Introduction

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$$u(t) = \begin{cases} u_o & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}.$$

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- Laplace transform of step:

$$U(s) = \mathcal{L}[u(t)] = \frac{u_o}{s}.$$

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Final value and static gain

Assume that the linear system $G(s)$ is stable, and that the input is a step

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The **static gain** of a stable system is $G(0)$

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Step response for first order systems

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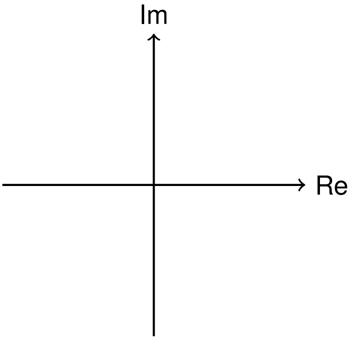

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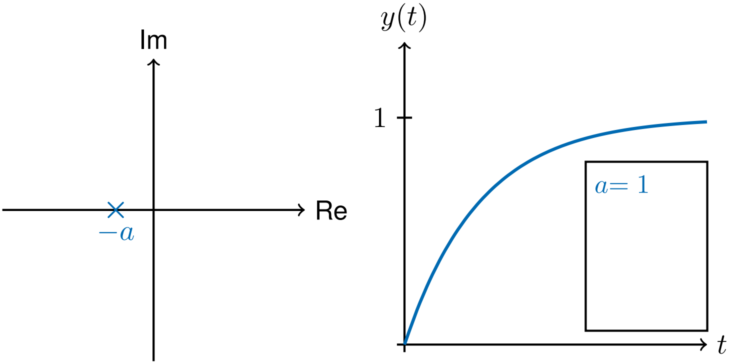

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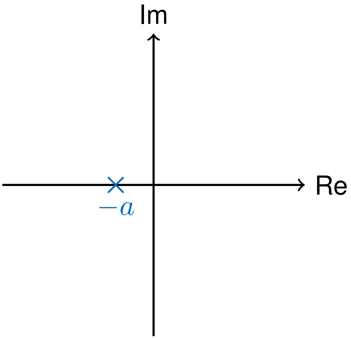

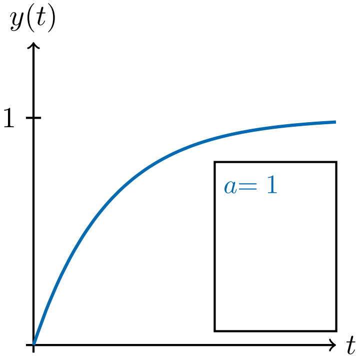

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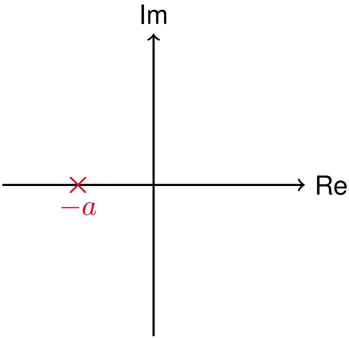

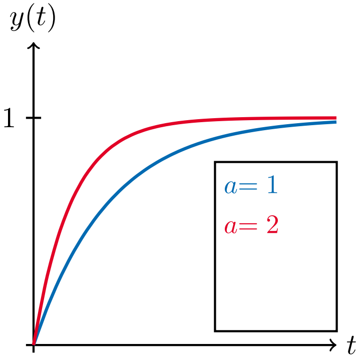

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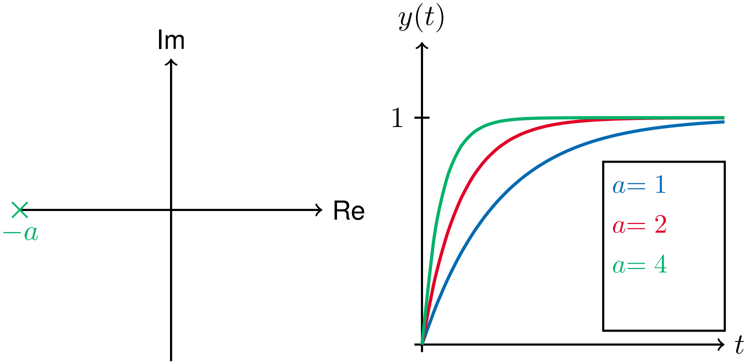

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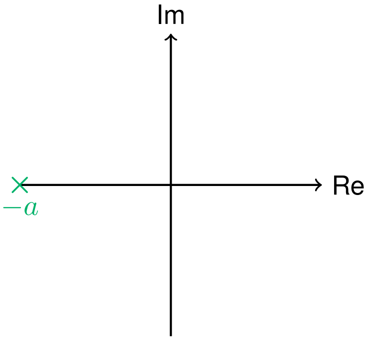

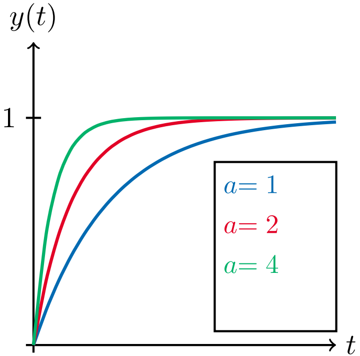

Longer the distance between the pole and the origin $\Rightarrow$ Faster system.

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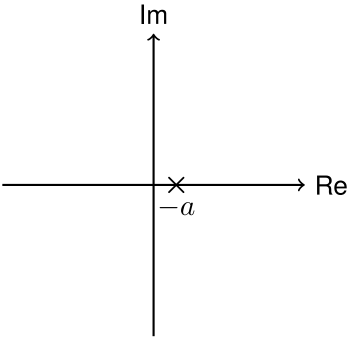

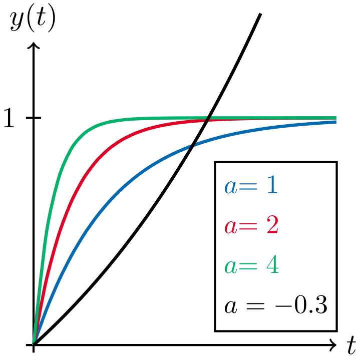

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Real and distinct poles

Consider a system $G(s) = \dfrac{a_2}{s^2 + a_1 s + a_2}$

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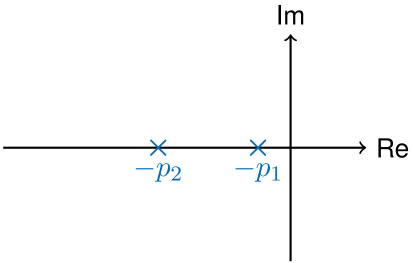

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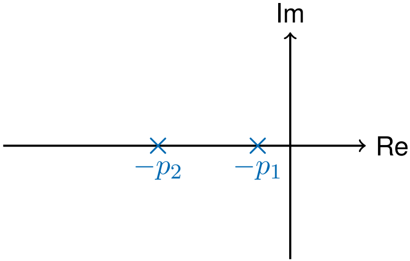

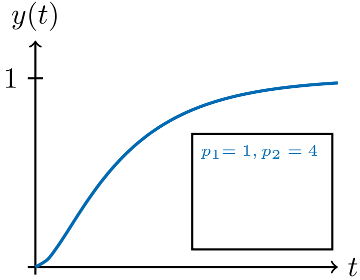

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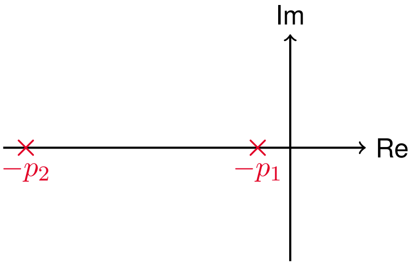

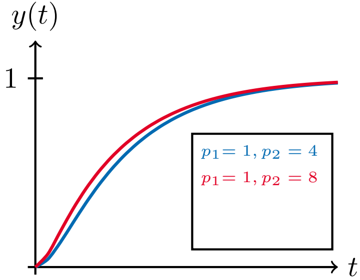

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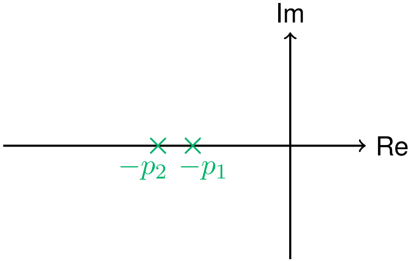

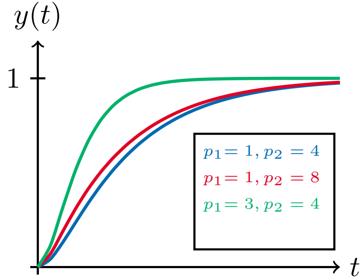

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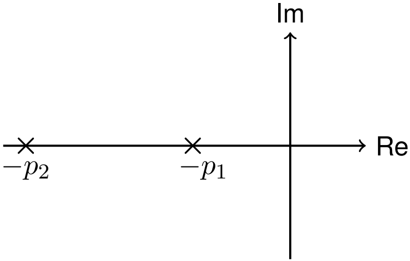

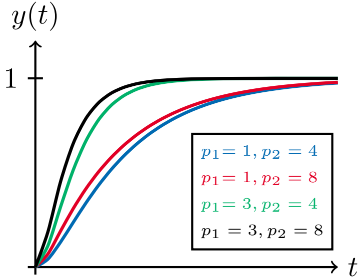

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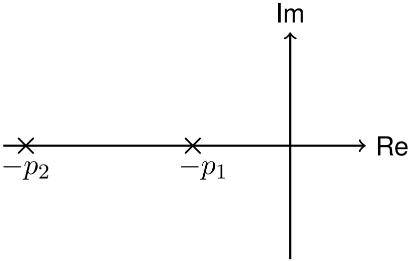

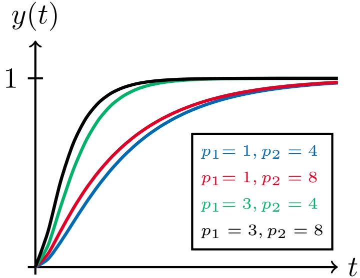

The pole closest to the origin dominates!

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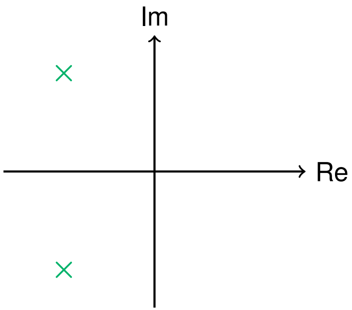

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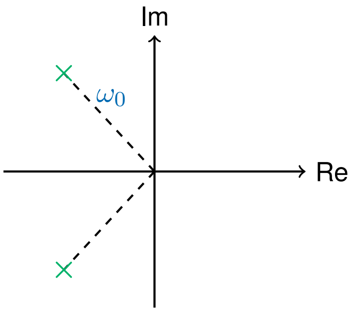

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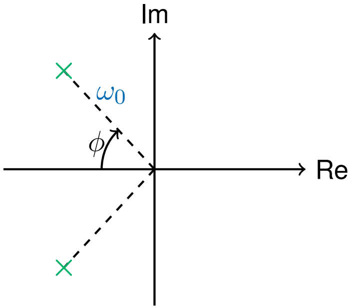

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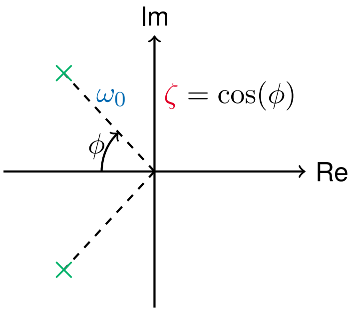

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Step response for second order systems

Complex conjugated poles

Consider the system $G(s) = \dfrac{a_2}{s^2 + a_1 s + a_2} = \dfrac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$.

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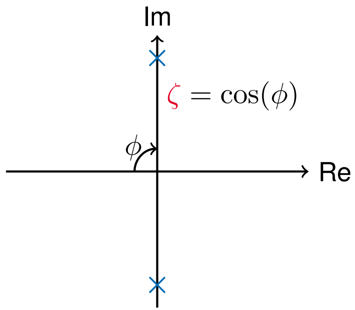

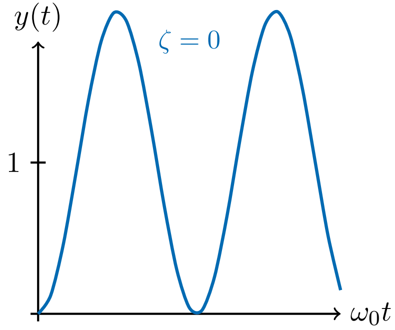

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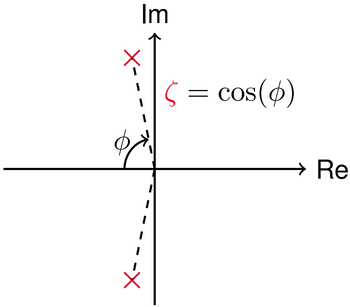

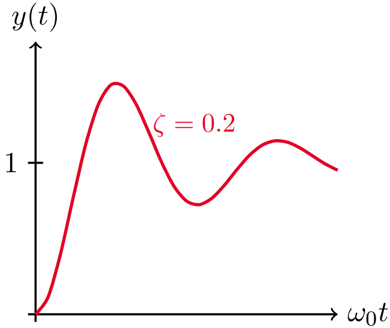

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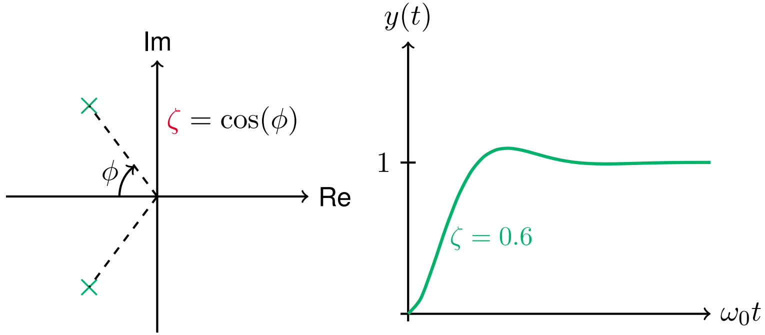

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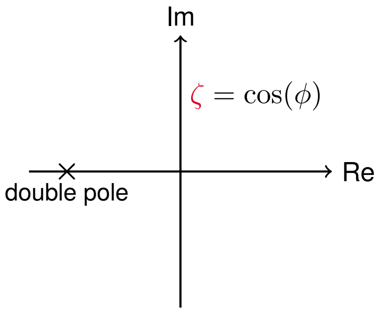

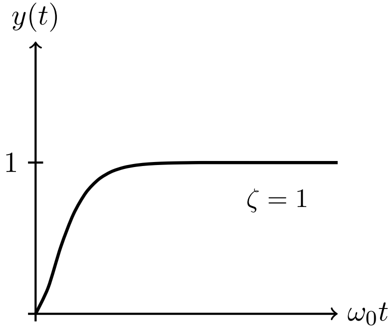

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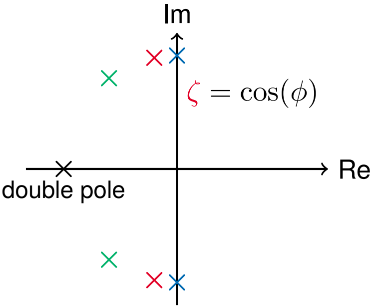

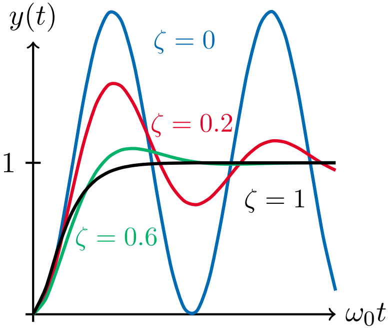

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Summary

- A linear system is stable if and only if all poles lie strictly in the left half-plane (has strictly negative real part).

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- The larger the imaginary part is relative to the real part of a pole, the more oscillations it gives to the step response.

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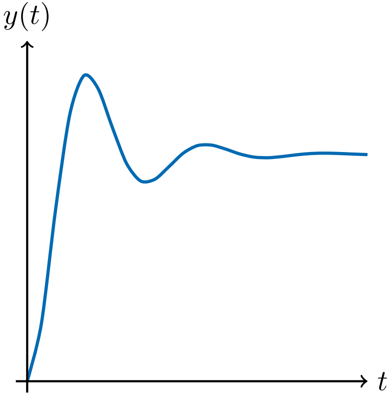
Step response

Specifications



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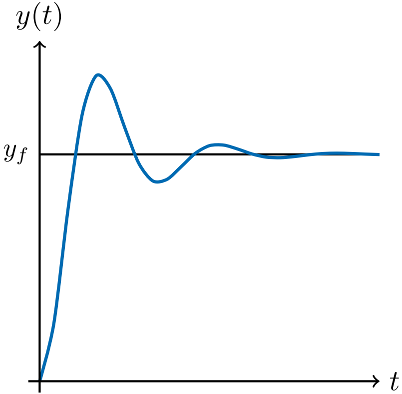

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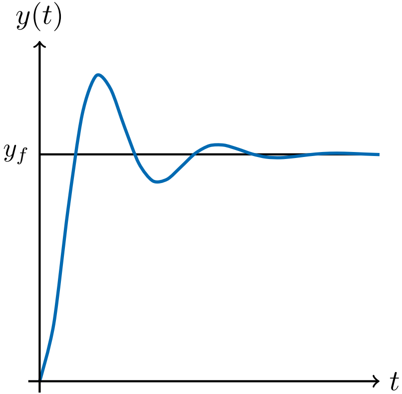

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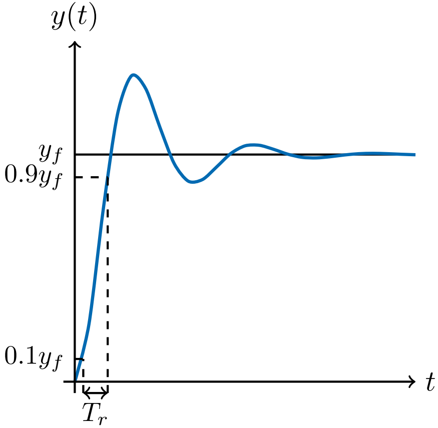

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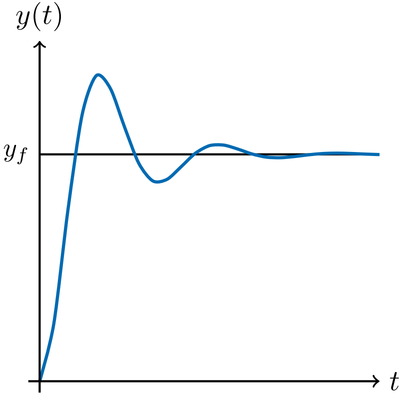

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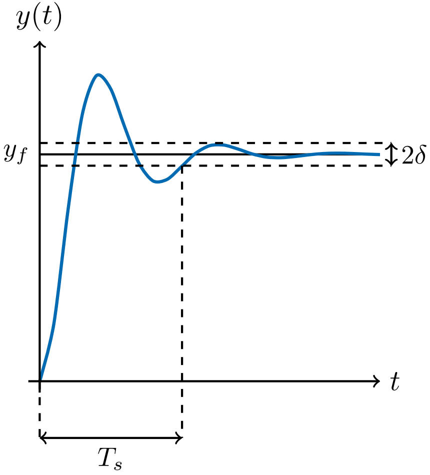

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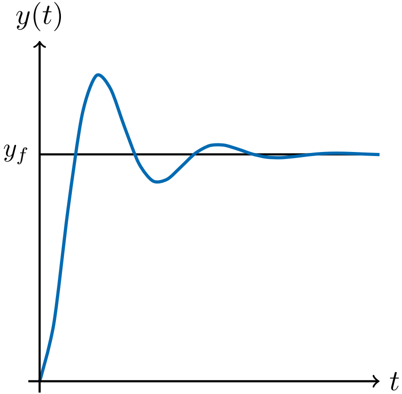

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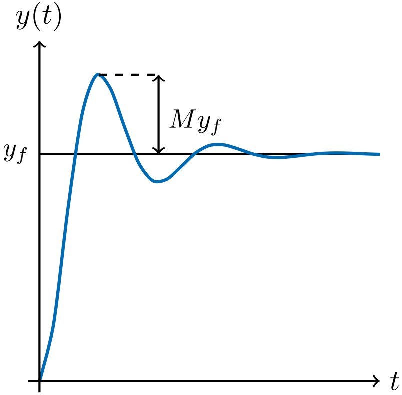

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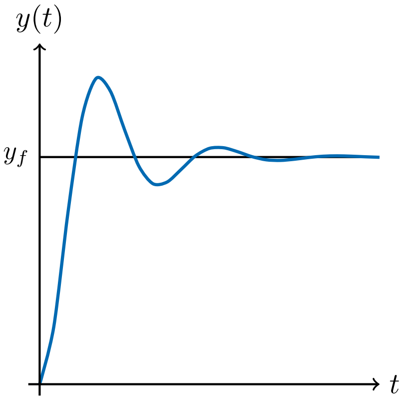

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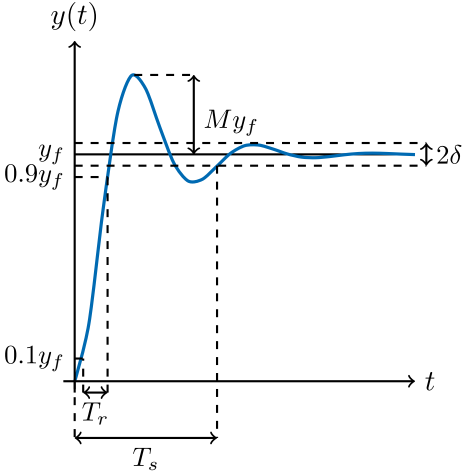

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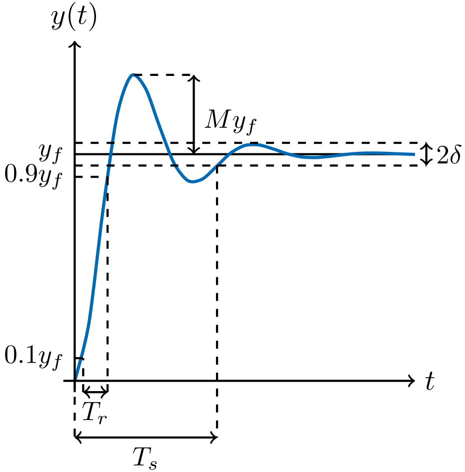

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Design criterion:

Choose the controller so that M, T_r and T_s are small enough.

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